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RATIONAL CONTRACTION CONDITIONS IN DISLOCATED QUASI-METRIC SPACE

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Abstract

In this article, we have set up a coupled fixed point hypothesis fulfilling objective contraction conditions in dislocated quasi-metric space. To approve our build up hypothesis and culminations we have give a model.

Keywords: *Conditions, Quasi-Metric*

Introduction

The idea of dislocated metric space was presented by Hitzler [1]. In such a space self distance between focuses need not be zero essentially. They additionally summed up the popular Banach contraction rule in dislocated metric space. Dislocated metric space assumes an essential part in geography, legitimate programming, software engineering, and electronic designing, and so forth In 2005, Zeyada, Hassan, and Ahmad [2] started the idea of complete dislocated quasi-metric space and summed up the aftereffect of Hitzler [1] in dislocated quasi-metric space. With the progression of time many papers have been distributed containing fixed point results for a solitary and a couple of mappings for various sorts of contractive conditions in dislocated quasi-metric space (see [3, 4, 5, 6]). In 2006, Bhaskar and Lakshmikantham [7] started the idea of coupled fixed focuses for non-straight contractions in somewhat requested metric spaces. Moreover, after crafted by Bhaskar and Lakshmikantham [7] coupled fixed point hypotheses have been set up by many creators in an alternate kinds of spaces (see [8, 9, 10]). In this paper, we have set up a coupled fixed point hypothesis fulfilling levelheaded contraction conditions with regards to dislocated quasi-metric space. A model is given in the help of our primary outcomes. All through the paper, R+ addresses the arrangement of non-negative genuine numbers.

Definition[2]. Let X be a non-empty set and let $d : X \times X \rightarrow R+$ be a function satisfying the conditions

d1) $d(x, x) = 0$;

d2) $d(x, y) = d(y, x) = 0$ implies $x = y$;

d3) $d(x, y) = d(y, x)$;

d4) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

On the off chance that d fulfills the conditions from d1 to d4, it is called metric on X. Assuming d fulfill conditions d2 to d4 then it is called dislocated metric (d-metric) on X and in the event that d fulfill conditions d2 and d4 really at that time it is called dislocated quasi-metric (dq-metric) on X. The pair (X, d) is called dislocated quasi-metric space. Obviously every metric space is a dislocated metric space however the opposite isn't really evident as clear from the accompanying model:

Example Let $X=\mathbb{R}^+$ define the distance function $d : X \times X \to \mathbb{R}^+$ by

$$d(x, y) = max\{x, y\}$$

Obviously, X is dislocated metric space yet not a metric space. Additionally every metric and dislocated metric space are dislocated quasi-metric spaces however the opposite isn't accurate.

Example. Let X = R define the distance function d : X × X → R+ by

$$d(x, y) = |x| \quad \text{for all } x, y \in X.$$

Clearly X is dq-metric space yet not a metric space nor dislocated metric space. In our fundamental work we will utilize the accompanying definitions which can be found in [2].

Definition. A sequence $\{x_n\}$ is called dislocated quasi convergent (dq-convergent) in X if for n ∈ N we have

$$\lim_{n\to\infty} d(x_n, x) = \lim_{n\to\infty} d(x, x_n) = 0.$$

In this case x is called dislocated quasi limit (dq-limit) of the sequence $\{x_n\}$.

Definition. A dq-metric space (X, d) is said to be complete if every Cauchy sequence in X converges to a point in X.

Definition[7]. An element (x, y) ∈ X2 is called coupled fixed point of the mapping T : X×X → X if T(x, y) = x and T(y, x) = y for x, y ∈ X.

Example Let X = R and T : X × X → X defined b

$$T(x_1, x_2) = \frac{x_1 x_2}{2}.$$

Here (0, 0) is the coupled fixed point of T.

The following well-known results can be seen in [2].

Lemma 1.1. Limit of a convergent sequence in dq- metric space is unique.

Theorem 1.2. Let (X, d) be a complete dq-metric space T : X → X be a contraction. Then T has a unique fixed point.

Remark

It is obvious that the following statement hold.

For real numbers a, b and c,

If a < b and c > 0. Then ac < bc.

2 Main Results

Hypothesis 2.1. Let (X, d) be a finished dislocated quasi-metric space. T: X × X → X be a persistent planning fulfilling the accompanying objective contractive conditions

$$d(T(x,y),T(u,v)) \le \alpha \cdot [d(x,u)+d(y,v)] + \beta \cdot \frac{d(x,T(x,y)).d(x,T(u,v))}{1+d(x,u)+d(y,v)} + \gamma \cdot \frac{d(x,T(x,y)).d(u,T(u,v))}{1+d(x,u)} \quad (2.1)$$

For all x, y, u, v ∈ X and α, β and γ are non-negative constants with 2(α + β) + γ < 1. Then T has a unique coupled fixed point in X × X.

Proof Let x0 and y0 are arbitrary in X, we define the sequences {xn} and {yn} as following,

$$x_{n+1} = T(x_n, y_n) \quad \text{and} \quad y_{n+1} = T(y_n, x_n) \quad \text{for } n \in N.$$

Consider

$$d(x_n, x_{n+1}) = d(T(x_{n-1}, y_{n-1}), T(x_n, y_n))$$

Now by (2.1) we have

$$d(x_n, x_{n+1}) \le \alpha \cdot [d(x_{n-1}, x_n) + d(y_{n-1}, y_n)] + \beta \cdot \frac{d(x_{n-1}, T(x_{n-1}, y_{n-1})).d(x_{n-1}, T(x_n, y_n))}{1 + d(x_{n-1}, x_n) + d(y_{n-1}, y_n)} + \gamma \cdot \frac{d(x_{n-1}, T(x_{n-1}, y_{n-1})).d(x_n, T(x_n, y_n))}{1 + d(x_{n-1}, x_n)}.$$

Using the definition of the sequences {xn} and {yn} we have

$$\le \alpha \cdot [d(x_{n-1}, x_n) + d(y_{n-1}, y_n)] + \beta \cdot \frac{d(x_{n-1}, x_n).d(x_{n-1}, x_{n+1})}{1 + d(x_{n-1}, x_n) + d(y_{n-1}, y_n)} + \gamma \cdot \frac{d(x_{n-1}, x_n).d(x_n, x_{n+1})}{1 + d(x_{n-1}, x_n)}.$$

Simplifying and using Remark 1 we have

$$< \alpha \cdot [d(x_{n-1}, x_n) + d(y_{n-1}, y_n)] + \beta \cdot d(x_{n-1}, x_{n+1}) + \gamma \cdot d(x_n, x_{n+1})$$

$$\le \alpha \cdot [d(x_{n-1}, x_n) + d(y_{n-1}, y_n)] + \beta \cdot [d(x_{n-1}, x_n) + d(x_n, x_{n+1})] + \gamma \cdot d(x_n, x_{n+1}).$$

Simplification yields

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$$d(x_n, x_{n+1}) \leq \frac{\alpha+\beta}{1-(\beta+\gamma)} d(x_{n-1}, x_n) + \frac{\alpha}{1-(\beta+\gamma)} d(y_{n-1}, y_n). \qquad (2.2)$$

Similarly we can show that

$$d(y_n, y_{n+1}) \leq \frac{\alpha+\beta}{1-(\beta+\gamma)} d(y_{n-1}, y_n) + \frac{\alpha}{1-(\beta+\gamma)} d(x_{n-1}, x_n). \qquad (2.3)$$

Adding (2.2) and (2.3) we have

$$[d(x_n, x_{n+1}) + d(y_n, y_{n+1})] \leq \frac{2\alpha+\beta}{1-(\beta+\gamma)} [d(x_{n-1}, x_n) + d(y_{n-1}, y_n).$$

Since $2\alpha + 2\beta + \gamma < 1$, so $h = \frac{2\alpha+\beta}{1-(\beta+\gamma)} < 1.$ 1. Therefore the above inequality becomes,

$$[d(x_n, x_{n+1}) + d(y_n, y_{n+1})] \leq h \cdot [d(x_{n-1}, x_n) + d(y_{n-1}, y_n).$$

Also

$$[d(x_n, x_{n+1}) + d(y_n, y_{n+1})] \leq h^2 \cdot [d(x_{n-2}, x_{n-1}) + d(y_{n-2}, y_{n-1}).$$

Similarly proceeding we have

$$[d(x_n, x_{n+1}) + d(y_n, y_{n+1})] \leq h^n \cdot [d(x_0, x_1) + d(y_0, y_1).$$

Since h < 1 taking limit n → ∞ we have

$$[d(x_n, x_{n+1}) + d(y_n, y_{n+1})] \to 0.$$

Implies

$$d(x_n, x_{n+1}) \to 0 \text{ and } d(y_n, y_{n+1}) \to 0.$$

Thus {xn} and {yn} are Cauchy sequences in complete dislocated quasi-metric space X. So there must exist w, z ∈ X such that

$$\lim_{n\to\infty} x_n = w \text{ and } \lim_{n\to\infty} y_n = z.$$

Also since T is continuous and T(xn, yn) = xn+1 so taking limit n → ∞. We have

$$\lim_{n\to\infty} T(x_n, y_n) = \lim_{n\to\infty} x_{n+1}$$

$$T(\lim_{n\to\infty} x_n, \lim_{n\to\infty} y_n) = \lim_{n\to\infty} x_{n+1}$$
$$T(w, z) = w.$$

Also from T(yn, xn) = yn+1. We can show that

$$d(z, w) = z.$$

Thus (w, z) ∈ X × X is the coupled fixed point of T in X.

Uniqueness. Let (w, z) and (w 0 , z 0) are two distinct coupled fixed points of T in X × X. Then by use of (2.1) we have

$$d(w,w)=d(T(w,z),T(w,z))\le\alpha\cdot[d(w,w)+d(z,z)]+$$
$$\beta\cdot\frac{d(w,T(w,z)).d(w,T(w,z))}{1+d(w,w)+d(z,z)}+\gamma\cdot\frac{d(w,T(w,z)).d(w,T(w,z))}{1+d(w,w)}$$
$$\le\alpha\cdot[d(w,w)+d(z,z)]+\beta\cdot\frac{d(w,w).d(w,w)}{1+d(w,w)+d(z,z)}+\gamma\cdot\frac{d(w,w)).d(w,w))}{1+d(w,w)}.$$

Using Remark 2.1 and then simplifying we have

$$d(w,w)\le(\alpha+2\beta+\gamma)d(w,w)+\alpha d(z,z). \qquad (2.4)$$

Similarly we can show that

$$d(z,z)\le(\alpha+2\beta+\gamma)d(z,z)+\alpha d(w,w). \qquad (2.5)$$

Adding (2.4) and (2.5) we have

$$[d(w,w)+d(z,z)]\le(2\alpha+2\beta+\gamma)[d(w,w)+d(z,z)].$$

Since 2α + 2β + γ < 1 so the above inequality is possible only if

$$[d(w,w)+d(z,z)]=0.$$

$$d(w,w)=d(z,z)=0. \qquad (2.6)$$

Now consider

$$d(w,w')=d(T(w,z),T(w',z')\le\alpha\cdot[d(w,w')+d(z,z')]$$
$$+\beta\cdot\frac{d(w,T(w,z)).d(w,T(w',z'))}{1+d(w,w')+d(z,z')}+\gamma\cdot\frac{d(w,T(w,z)).d(w',T(w',z'))}{1+d(w,w')}$$
$$\le\alpha\cdot[d(w,w')+d(z,z')]+\beta\cdot\frac{d(w,w).d(w,w')}{1+d(w,w')+d(z,z')}+\gamma\cdot\frac{d(w,w).d(w',w')}{1+d(w,w')}.$$

Now using (2.6) we have the following

$$d(w,w')\le\alpha\cdot[d(w,w')+d(z,z')]. \qquad (2.7)$$

By following similar procedure we can get

$$d(z,z')\le\alpha\cdot[d(z,z')+d(w,w')]. \qquad (2.8)$$

Adding (2.7) and (2.8) we have

$$d(w,w^{'})+d(z,z^{'})\leq 2\alpha\cdot[d(w,w^{'})+d(z,z^{'})].$$

Since 2α < 1 so the above inequality is possible only if

$$d(w,w^{'})+d(z,z^{'})=0.$$

Which implies that?

$$d(w,w^{'})=d(z,z^{'})=0.$$

Implies

$$w=w^{'} \text{ and } z=z^{'}.$$

Hence

$$(w,z)=(w^{'},z^{'}).$$

Thus coupled fixed point of T in X × X is unique. We deduce the following corollaries from Theorem 2.1.

Corollary 2.2. Let (X, d) be a complete dislocated quasi-metric space. T : X × X → X be a continuous mapping satisfying the following rational contractive conditions

$$d(T(x,y),T(u,v))\leq\alpha\cdot[d(x,u)+d(y,v)]+\beta\cdot\frac{d(x,T(x,y)).d(x,T(u,v))}{1+d(x,u)+d(y,v)}$$

For all x, y, u, v ∈ X and α, β are non-negative constants with 2(α + β) < 1. Then T has a unique coupled fixed point in X × X.

Corollary 2.3. Let (X, d) be a complete dislocated quasi-metric space. T : X × X → X be a continuous mapping satisfying the following rational contractive conditions

$$d(T(x,y),T(u,v))\leq\alpha\cdot[d(x,u)+d(y,v)]+\beta\cdot\frac{d(x,T(x,y)).d(u,T(u,v))}{1+d(x,u)}$$

For all x, y, u, v ∈ X and α, β are non-negative constants with 2α+β < 1. Then T has a unique coupled fixed point in X × X.

Corollary 2.4. Let (X, d) be a finished dislocated quasi=metric space. T: X × X → X be a persistent planning fulfilling the accompanying sane contractive conditions

$$d(T(x,y),T(u,v))\leq\alpha[d(x,u)+d(y,v)]$$

For all x, y, u, v ∈ X and α > 0 with 2α < 1. Then T has a unique coupled fixed point in X ×X.

Example. Let X = [0, 1]. Define d : X × X → R+ by

$$d(x,y)=|x-y|+|x|$$

for all x, y ∈ X. Then (X, d) is a complete dislocated quasi-metric space. Define a continuous self-map T : X × X → X by T(x, y) = 1 6 xy for all x, y ∈ X. Since

$$|xy - uv| \leq |x - u| + |y - v| \text{ and } |xy| \leq |x| + |y|.$$

Hold for all x, y, u, v ∈ X. Then

$$d(T(x, y), T(u, v)) = |\frac{1}{6}xy - \frac{1}{6}uv| + |\frac{1}{6}xy|$$

$$\leq \frac{1}{6}(|x - u| + |y - v|) + \frac{1}{6}(|x| + |y|)$$

$$\leq \frac{1}{6}(|x - u| + |y - v| + |x| + |y|)$$

$$\leq \frac{1}{3}[(|x - u| + |x|) + (|y - v| + |y|)]$$

$$d(T(x, y), T(u, v)) \leq \alpha \cdot [d(x, u) + d(y, v)].$$

So for α = 1 3 and β = γ = 0 all the conditions of Theorem 2.1 are satisfied having (0, 0) ∈ X×X is the unique coupled fixed point of T in X × X.

CONCLUSION

We have studied existence of coincidences and fixed points of non–expansive type conditions satisfied by single-valued and multi-valued maps which are wide applicable in many branches of engineering and science. Also we proved fixed points for weakly contractive self-maps in complete Metric Space which are our new contribution in research. In this chapter, some common fixed point theorems are proved related to complete and compact metric space and there is enough scope for further work in this direction. Further we derive couple of common fixed point theorems with different sufficient conditions and discuss generalization of some known results help the researchers to improve and expand results. We proved common fixed point and extended results in commuting maps, weakly commuting maps and compatible maps which is our new research work in fixed point theory. In this chapter, we considered as a continuation of remarkable works of various authors. We studied fixed point theorems in quasi – pseudo metric space, dislocated quasi – metric space and to introduce new fixed point theorems. In the light of these works, it may be possible to extend some of the results in this paper which are important in computer forensics and cryptography in securing data and information.

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